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An Investigation of Secondary Teachers' Understanding and Belief on Mathematical Problem Solving

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Abstract. Weaknesses on problem solving of Indonesian students as reported by recent international surveys give rise to questions on how Indonesian teacher bring out idea of problem solving in mathematics lesson. An explorative study was undertaken to investigate how secondary teachers who teach mathematics at junior high school level understand and show belief toward mathematical problem solving. Participants were teachers from Surabaya, Sidoarjo, Gresik and Mojokerto. There were 45 state teachers and 25 private teachers. Data was obtained through questionnaires and written test. The results of this study point out that actually the teachers understand problem solving as pedagogical knowledge well as indicated by high score of observed teachers' responses showing understanding on problem solving as instruction as well as implementation of problem solving in teaching practice although they less understand on problem solving content knowledge such as problem solving strategies and meaning of problem itself. Regarding teacher's difficulties, teachers admitted to most frequently failed in (1) determining a precise mathematical model or strategies when carrying out problem solving steps which is supported by data of test result that revealed transformation error as the most frequently observed errors in teachers' work and (2) choosing suitable real situation when designing context-based problem solving task. Meanwhile, analysis of teacher's beliefs on problem solving shows that teachers tend to view both mathematics and how students should learn mathematics as body static perspective, while they tend to believe to apply idea of problem solving as dynamic approach when teaching mathematics.

Keywords: problem solving, beliefs in mathematics, teachers' understanding.

1. Introduction

It has already been agreed that problem solving is an essential issue deeply discussed in mathematics education in recent decades since its practical role to the individual and society. Problem-solving as one of five standard competences in mathematics mentioned by National Council of Teachers of Mathematics [1] not only develops individuals' conception about aspects of mathematics, but also it helps to adapt to various problems in many aspects of their lives. NCTM [1] also recommended that

problem solving be the focus of mathematics teaching because, they say, it encompasses skills and functions which are an important part of everyday life.

This issue brings a variety of responds which regards to both story of success and difficulties in applying it within practical mathematics teaching and learning in many countries. It is generally known that mathematics curriculum in some countries can be ascertained put problem solving as a main component, such as in Singapore [2,3], United States [2], England [2], Australia [2], Finland [4], and Indonesia [5]. The success story, for example, was reported by [6] which showed that problem-solving, based on open tasks in specific topic is feasible and effective in the classroom. Students are challenged and actively involved in cognitive and physical activities as well as in interesting discoveries. In Finland, the country where often puts their students in top rank in international survey like PISA [7], problem solving has been a part of the Finnish curriculum for a couple of decades but still it has not found its place in the classroom reality [9]. Singapore curriculum, on the other hand, encourages teachers to reduce the content taught through direct teaching but instead engage students in meaningful activities so that they use knowledge to solve problems and whilst solving problems extend their knowledge through inquiry [3].

However, the issue of mathematical problem solving in Indonesia, especially, has not yielded a satisfactory result although it has been becoming one of curriculum contents of school mathematics since year 1968 [5]. [60] shows that the students' ability to solve mathematical problems is still weak as pointed out by the results of international studies on education, such as TIMSS and PISA. Result of TIMSS 2011 [8] participated by secondary students grade 8 reported that Indonesia ranked 38th out of 42 countries with score 386 of center point of TIMSS of 500, while the latest PISA report in 2012, participated by 15-year old secondary students, Indonesia was reported [20] get rank of 64 out of 65 countries. In the latter study, even almost all Indonesian students (98.5%) were only able to reach level 3 of 6 levels of the task examined [7,10]. Research on Indonesian students' performances on problem solving, particularly, also supported this issue. The study of Siswono, Abadi, and Rosyidi [11] to the fifth grade students as many as 202 students from five elementary schools in Sidoarjo showed that students' ability in solving problems (especially the open-ended problem) is still low as indicated by the data that the students' ability to solve problems that show fluency is only 17.8%, novelty 5.0%, and flexibility 5.4%. This result is also supported by studies of Kohar & Zulkardi [12] and Wijaya, van den Heuvel-Panhuizen, Doorman, & Robitzsch [13] each of which qualitatively investigate the extend to which secondary students perform their mathematical problem solving stages on context-based task which point out that the student participants mostly failed in early stages of contextual problem solving steps, namely devising strategies to transform contextual situation into precise mathematical model.

This condition is supposed to be influenced by various factors such as students' background regarding on mathematics performance, the available of learning resources, teacher beliefs, the ability of teachers, facilities, learning process, as well as national educational policies. Teacher as crucial factor in the development of students' learning has role to build students' understanding on applying idea of mathematical problem solving in real life. Therefore, research on teacher's [52] is important to be carried out, for instance, on their understanding [1] in problem solving, their content and pedagogical knowledge and skills, and their beliefs. The challenge for mathematics teachers is to foster mathematical learning, and the key media for pedagogical interaction between teacher and students is the tasks in which the students engage. This is what Sullivan, et al [6] stated as the essence of teaching problem solving.

Hence, this issue gives rise to the questions of how Indonesian teachers bring out idea of problem solving in their understanding related to both content and pedagogical knowledge as well as how they experience difficulties, and how are their beliefs on this issue. As an initial step to address these questions, this present study [65] aims to investigate secondary teachers' understanding and belief on mathematical problem solving. Therefore, the following research questions were addressed:

1. How did secondary teachers understand mathematical problem solving regarding problem, problem solving as instruction, problem solving steps, problem solving strategies, and

- instructional practice of problem solving as well as level of their performance on problem solving task?
2. What are secondary teachers' difficulties which are regarded to understand mathematical problem solving?
 3. How are secondary teachers' beliefs on mathematical problem solving?

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2. Theoretical Background

2.1. Teachers' understanding on the nature of problem solving

Teaching problem solving needs some understandings which are related to some points of knowledge. Chapman [14] mentioned three types of knowledge for teaching problem solving: problem solving content knowledge, pedagogical problem solving knowledge, and affective factors and beliefs. (see table 1). Structuring some knowledge within this table was then confirmed as follows. First, figuring out what it means by problem. Chapman [14] argued that teachers should understand problems based on their structure and purpose in order to make sense of how to guide students' solutions including understanding on types of tasks, such as cognitively demanding tasks; multiple-solution tasks; tasks with potential to occasion/promote mathematical creativity in problem solving; demanding problems that generally allowed for a variety of problem-solving strategies; rich mathematical tasks, and particularly open-ended problems. Second, understanding problem solving in instruction which means teachers are encouraged to foster their students in completing problem solving steps precisely.

Table 1. Knowledge needed in understanding problem solving

Type of knowledge	Knowledge	Description
Problem solving content knowledge	Mathematical problem solving proficiency	Understanding what is needed for successful mathematical problem solving
	Mathematical problems	Understanding of the nature of meaningful problems; structure and purpose of different types of problems; impact of problem characteristics on learners
	Mathematical problem solving	Being proficient in problem solving Understanding of mathematical problem solving as a way of thinking; problem solving models and the meaning and use of heuristics; how to interpreting students' unusual solutions; and implications of students' different approaches
	Problem posing	Understanding of problem posing before, during and after problem solving
Pedagogical problem solving knowledge	Students as mathematical problem solvers	Understanding what a student knows, can do, and is disposed to do (e.g., students' difficulties with problem solving; characteristics of good problem solvers; students' problem solving thinking)
	Instructional practices for problem solving	Understanding how and what it means to help students to become better problem solvers (e.g., instructional techniques for heuristics/strategies, metacognition, use of technology, and assessment of students' problem solving progress; when and how to intervene during students' problem solving).
Affective factors and beliefs		Understanding nature and impact of productive and unproductive affective factors and beliefs on learning and teaching problem solving and teaching

Third, coming up with idea of instructional practices for problem solving, teachers need to consider a series of activities which give students opportunity to solve problems which needs challenging complex thinking and logical reasoning. The activity depends on how the teacher's ability to prepare a problem. Crespo & Sinclair [in 15] explained that teachers who are able to create questions in the initial situation will be more successful learning rather than the teacher who asked the problems simultaneously. Thus, teaching for problem solving, teachers should be proficient in as well as understand the nature of it in order to teach it effectively. In general, this categorization appears to give insight on emerging teachers' conception of problem solving theoretically to teachers' actual teaching practically.

In addition to the specific issue related to problem solving content knowledge, teachers should also be proficient to deal with a variety of problem solving task, such as completing problem solving steps as well as applying problem solving strategies on various types of mathematical task. Level of understanding regarding this issue were discussed by some frameworks, such as Polya's problem solving step [16], and some error analysis guidelines on performing mathematical tasks developed by, for instance, Wijaya et al [17] who adapted from three main frameworks, namely Newman's error [18], Blum and Less' modelling stages [as cited in 19], and PISA's mathematization stages [20], and Kohar & Zulkardi [12] who adapted from Valley, Murray, & Brown [21] and PISA's mathematical literacy [22]. To figure out the possibility of those frameworks are applied to investigate level of teachers' proficiency in solving a variety of problem solving task, the following table shows characteristics of some of those.

Table 2. Comparing frameworks on performance level in solving mathematical tasks

Polya's Problem Solving Step [16]	Newman Analysis [18]	PISA's Mathematical Literacy [22]
<i>Understanding the problem:</i> identify the unknown, data, and condition related to the information given in the problem.	<i>Reading:</i> recognize of words and symbols <i>Comprehension:</i> understand the meaning of a problem	<i>Formulate:</i> recognise and identify opportunities to use mathematics and then provide mathematical structure to a problem presented in some contextualised form
<i>Devising a plan:</i> find the connection between data and the unknown, consider auxiliary problem if an immediate connection cannot be found, and finally obtain a plan of the solution	<i>Transformation:</i> transform a word problem into an appropriate mathematical problem	
<i>Carrying out the plan</i>	<i>Process skills:</i> perform mathematical procedures	<i>6</i> <i>Apply:</i> apply mathematical concepts, facts, procedures, and reasoning to solve mathematically-formulated problems to obtain mathematical conclusions
<i>Looking back:</i> examine the solution obtained	<i>Encoding:</i> represent the mathematical solution into acceptable written form	<i>6</i> <i>Interpret/evaluate:</i> reflect on mathematical solutions, results, or conclusions and interpret them in the context of real-life problems.

In general, from table 2 we can notice that element of steps among those three frameworks corresponds each other. Specifically, steps of understanding problem and devising strategies, simultaneously, has likely similar idea with steps of reading, comprehension, and transformation in

Newman analysis, while this idea also appear on mathematical literacy, i.e. formulate. As early stages in solving mathematical task, they end up with determining precise mathematical model or strategies before performing further steps of solving problem. Likewise, each idea of carrying out step in Polya's process, process skill in Newman, and employ in PISA's mathematical literacy deals with undertaking mathematical procedure to find mathematical results, such as performing arithmetic computations, solving equations, making logical deductions from mathematical assumptions, performing symbolic manipulations, or extracting mathematical information from tables and graph. Furthermore, the last step of Polya's, i.e. looking back, corresponds to final stage of Newman analysis, i.e. encoding and PISA's mathematical literacy, i.e. interpretation. The idea of this stage is interpreting the mathematical result to the initial problem such as checking the reasonableness of the answer or considering other strategies and solution of the problem. The difference, obviously, only appear on the type of the tasks examined where PISA's mathematical literacy specifies on contextual task [22], while Polya and Newman respectively deals with general mathematical problem [16] and written mathematical task [18]. Considering these views, Newman's error categories can be used to analyze teachers' level of performance in solving context-based mathematical problem solving tasks.

The typical teaching of problem solving should also need to be considered as important parts of understanding instructional practices for problem solving pedagogical knowledge. As examples, typical learning in Japan [23] consists of (1) discuss the previous lesson, (2) presents a problem, (3) ask students work individually or in groups, (4) discussion of problem-solving method, and (5) provides a summary and discussion of an important point. Shimizu [23] explains that underline and summarizes the activities or "Matome" has the function (1) underline and summarize the main points, (2) to encourage reflection of what we have done, (3) determine the context to recognize new concepts or terms based on previous experience, and (4) make the connection (connection) between the new and the previous topic. Similarly, teachers in Finland as reported by Koponen [24] in his study try out problem solving lesson by firstly introducing the problem, then working on the problem in pairs or in small groups, instructing the individual students on their solutions, and the final whole-classroom discussion.

Based on the above discussion it is necessary to know actually how teachers understand the nature of problem solving that includes problem itself, problem-solving, problem-solving strategies, problem solving approach brought into classroom practical teaching, and posing problem solving task. Besides, it is also necessary to look into teachers' performance in solving problem solving task.

2.2. Teachers' beliefs on mathematical problem solving

Mathematical beliefs, as Raymond said, are regarded as "personal judgments about mathematics formulated from experiences in mathematics"[25]. They play role as prerequisite development of problem solving itself [26]. Regarding their categorization related to other interrelated fields, Weldeana & Abraham [27] summarized frameworks of teachers' mathematical belief systems into smaller subsystems, including beliefs about the following: (a) the nature of mathematics, (b) the actual context of mathematics teaching and learning, and (c) the ideal context of mathematics teaching and learning. These subsystems are wide-ranging, including, for examples, teachers' views on mathematical knowledge; the role of learners and learning; the role of teachers and teaching; and nature of mathematics activities. This categorization was also conceptualized by Ernest [28] who described three views of nature of mathematics: the instrumentalist view, the platonist view, and the problem solving view. The instrumentalist believes mathematics is useful and collects unrelated facts, rules and skills. The platonist views mathematics as a consistent, connected and objective structure, which means mathematics is a unified body of knowledge that is discovered, not created. The problem solving view sees mathematics as a dynamically organized structure located in a social and cultural context.

In attempt to simplify these views, Beswick [29] has tried to make connections among the nature of mathematics, mathematics learning, and mathematics teaching as follows.

Table 3. Connections among belief about mathematics, mathematics teaching, and mathematics learning

Beliefs about the nature of mathematics	Beliefs about mathematics teaching	Beliefs about mathematics learning
Instrumentalist	Content focussed with an emphasis on performance	Skill mastery, passive reception of knowledge
Platonist	Content focussed with an emphasis on understanding	Active construction of understanding
Problem solving	Learner focussed	Autonomous exploration of own interests

The relationship between teachers' beliefs about mathematics and its teaching and learning practice, has been investigated in many studies. Ernest [in 30] claimed that a teacher's personal view of mathematics underpins belief in the teaching and learning of mathematics. In line with this claim, Schoenfeld [30] stated that the teacher's sense of the mathematical enterprise determines the nature of the classroom environment that the teacher creates. Thus, beliefs influence teaching and learning practice. Other study, Ruthven's [31] study, for instance, argued that teachers need to broaden their perspective about ability and quality in mathematical learning which is probably more easily changed by changing practical teaching in classroom first, as the teachers' understanding of mathematics teaching and learning. Thus, teaching and learning practice influence teacher's belief.

Studies on how deep mathematics teacher develop beliefs on mathematics and its practice in teaching and learning were then investigated by some researchers. A study of Zhang and Sze [32] comparing preservice teachers' belief on mathematics in China and Thailand, for instance, revealed that generally teachers participating in this study believe that mathematics is about thinking, logic, and usefulness, rather than a subject of calculableness and preciseness. Regarding their belief on mathematics teaching and learning, the results showed that Chinese preservice teacher's beliefs are more like constructivist. Beswick [33] in his study on two mathematics teacher's view on the nature of mathematics and mathematics as school subject regarding on the three views: platonist, instrumentalist, and problem solving, suggested that more attention needs to be paid to the beliefs about the nature of mathematics that the teachers have constructed as a result of the cumulative experience of learning mathematics. Thus, we get insight on how important these views offer opportunities for teachers to rethink their own beliefs and get to know more about teaching practices.

These beliefs, as Schoenfeld [30] argued, give impact on students' belief in learning mathematics which then obviously influence their mathematics performances. The cause, for instance, is that teachers rely on established beliefs to choose pedagogical content and curriculum guidelines [e.g. 34]. If teachers tend to believe mathematics as a set of tools that contain facts, rules, and skills, the lesson will likely to be centered on teachers instead of students [35]. Furthermore, The importance of students 'and teachers' beliefs about the role of problem solving in mathematics is a prerequisite development of problem solving itself [37]. Romberg in [36] shows the relationship of elements in the teaching of mathematics as follows.

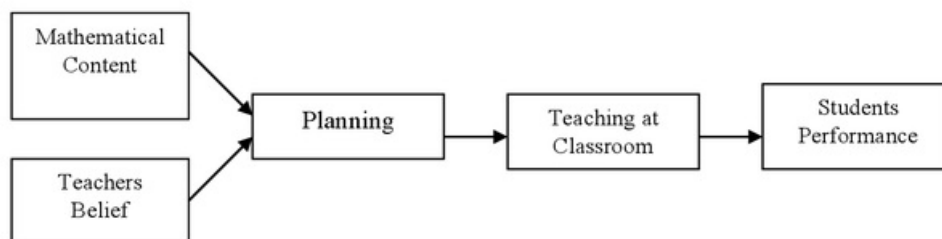


Figure 1. Relationship among elements of teaching mathematics

This view illustrates the importance of types of teacher beliefs which are needed to be investigated as attempt to improve teachers' proficiency dealing with problem solving.

3. Methods

This is a descriptive explorative research which aims to explore teachers' understanding on mathematical problem solving and their belief.

3.1. Participants

Participants are secondary teachers who has minimum a bachelor degree, have taught more than 5 years, from four cities: Surabaya, Sidoarjo, Gresik, and Mojokerto. There were 25 private teachers and 45 state involved in this study.

3.2. Data collection

Data were collected from questionnaire and problem solving task. The questionnaire consists of 21 questions. Each item provided 4 to 17 selection options. Teacher can choose more than one option. To explore the understanding of teachers to problem solving, there are 15 items provided which are categorized into 7 groups: problem, open ended problem, problem solving as instruction, problem solving steps, problem solving strategies, and instructional practice of problem solving, and designing problem solving task. Other three items are categorized to identify the difficulties of teachers in problem solving while the other three items are categorized to explore teacher beliefs. Meanwhile, the problem solving tasks were designed to explore teachers' understanding regarding contextual tasks which do not likely need any higher prerequisite formal mathematical knowledge. See the tasks at appendix. The following is the description on the tasks' demands.

Table 4. Description of problem solving tasks

No	Unit	Description	Source
1	Futsal score	Interpret information about a score achieved by a futsal team in a tournament which is implicitly stated from the information given in a table	Modified version from Kohar and Zulkardi [12]
2	World online mathematics contest	Formulate a mathematical model to determine a perfect time to hold an online mathematics contest participated by students from different countries in different continents	Developed by authors

The difference between those two task, particularly, appears on the most dominant stages which are likely more needed to be performed when solving the task. Here, task 1 needs more performance on final stage of problem solving, i.e. interpreting mathematical result to initial problem, while task 2, in opposite, demands more likely performance on early stages, i.e. from understanding the task to devising strategies or mathematical model of the task.

3.3. Data analysis

Descriptive analysis on investigating teachers' understanding and belief was carried out by using score given on each group of questions. Each option on a question has score either 1 (not understanding), 2 (partial understanding), or 3(full understanding). As an example, we give one question including its options and its score as follows.

An open-ended mathematics task is the task which...

- A. contains open sentences (score 1)
- B. produces a variety of strategies to find out the solutions (score 3)
- C. gives an open opportunity to persons who want to solve (score 1)
- D. contains higher level of difficulty so that needs higher mathematical skill as well (score 2)
- E. has more than one solutions (score 3)
- F. has opportunity to be developed into other type of tasks by changing information or requirements from the solved task (score 2)

The score varies to show level of understanding from 1.00 (do not understand), to 3.00 (fully understand), while the other scores varies to show level of beliefs on mathematical problem solving from 1.00 (platonist view/as tool) to 3.00 (problem solving view). The score is given to each participant on each question based on the following formula.

$$\text{Score (S)} = \frac{\text{obtained total score}}{\text{number of chosen options}}$$

In detail, we have developed a guideline to categorize these levels as shown by the following table.

Table 5. Scoring category level of teachers' understanding and beliefs

Score (S)	Level of understanding on mathematical problem solving	Level of beliefs toward mathematical problem solving
$1.00 \leq S < 1.67$	not understand (NU)	as tool/instrumentalist view
$1.67 \leq S \leq 2.33$	partially understand (PU)	as body static/platonist view
$2.33 < S \leq 3.00$	fully understand (FU)	as dynamic/problem solving view

Regarding teachers' performance on problem solving task, we used framework on investigating individual's performance based on stages of mathematical modelling adapted from Wijaya [13], and Kohar [12] since the task is in the category of context-based problem solving task. This is shown as follows.

Table 6. Level of teachers' performance on problem solving task

Type of responses	Sub-type	Codes	Explanation
Comprehension error	Misunderstanding the instruction	C-1	Teacher incorrectly interpreted what they were asked to do.
	Error in selecting information	C-2	Teacher was unable to differentiate between relevant and irrelevant information (e.g. using all information provided in a task, neglecting relevant information, or adding other unrelated information not given in the task) or was unable to gather required information which was not provided in the task.
Transformation/ devising strategies error	Procedural tendency	T-1	Teacher tended to use directly a mathematical procedure (such as formula, algorithm) without analyzing whether or not it was needed
	Taking too much account of the context	T-2	Teacher's answer only referred to the context/real world situation without taking the perspective of the mathematics
	Wrong mathematical operation/concept	T-3	Teacher used mathematical procedure/concepts which are not relevant to the tasks
Mathematical Processing error	Algebraic error	P-1	Error in solving algebraic expression or function.
	Arithmetical error	P-2	Error in calculation.
	Measurement error	P-3	Teacher could not convert between units (e.g. from hour to minute)
	Unfinished answer	P-4	Teacher used a correct formula or procedure, but she/he did not finish it.
Interpretation error		I	Teacher was unable to correctly interpret and validate the mathematical solution in terms of the real world problem. This error was reflected by an impossible or not realistic answer.
Full understanding	Arithmetical approach	F-1	Teacher performed complete and correct steps of solving the task by applying arithmetical approach dominantly (e.g. applying standard

Type of responses	Sub-type	Codes	Explanation
			arithmetic operation such as adding, subtracting, multiplying, or dividing number) to get solution
	Algebraic approach	F-2	Teacher performed complete and correct steps of solving the task by applying algebraic approach dominantly (e.g. building algebraic equation, manipulating algebraic form) to get solution
Unknown		U	Type of response could not be coded since its limited information from teacher's work

4. Results

4.1 Teachers' understanding on problem solving

4.1.1 Teachers' understanding in questionnaire results

There were seven categories of questions which were tested to measure teachers's understanding on problem solving. Each category could contain more than one questions. For instance, category of problem solving strategies contained two questions, while category of experience in designing problem solving tasks contained three questions. Table 7 shows teachers' average score on the questionnaire.

Table 7. Teachers' understanding on mathematical problem solving

Category of teachers' understanding on problem solving	Score			Interpretation
	Private	State	Total	
Meaning of problem	2.17	2.46	2.36	FU
Open-ended problem	2.65	2.80	2.75	FU
Problem solving as instruction	2.84	2.96	2.91	FU
Problem solving steps	2.41	2.50	2.47	FU
Problem solving strategies	1.89	1.80	1.83	PU
Implementation of steps and strategies of problem solving in teaching	2.43	2.45	2.44	FU
Experience in designing problem solving task	2.66	2.63	2.64	FU

Table 7 shows that the lowest score of understanding problem solving appears on category problem solving strategy (1.83) which means teachers did not really understand toward this group of questions. In giving response on questions related to problem solving strategy, data show that most teachers chose wrong options related to type of problem solving strategy should be applied on a given information on the question. Moreover, teacher did not really show good understanding on the meaning of problem as indicated by its score, i.e., 2.36, which could be interpreted as low score of full understanding based on scoring category on table 5. However, higher score appears on category problem solving as instruction (2.91) and designing problem solving task (2.64), both 55 which are related to practical knowledge of problem solving. It shows even though teachers are aware of the importance of problem solving as the focus of learning but there are still weaknesses in selecting a task question as a problem and solution strategies. Thus, regarding Chapman's category of knowledge needed to understand problem solving, we then could note that the teachers had relatively better understanding on pedagogical problem solving knowledge rather than problem solving content knowledge.

4.1.2 Teachers' understanding in performing problem solving task

In total, we had 140 possible responses (number of tasks done by all teachers in total) which included 35 correct responses (25%), 80 incorrect responses (57.15%), i.e., no credit or partial credit, and 25 missing responses (17.85%). Each incorrect response had an opportunity to be coded by more than one sub-type code since its different errors found from this response. For instance, a response could be coded as mathematical processing error subtype algebraic error (P-1) and interpretation simultaneously (I). Thus, the total number of responses was no longer 140 items, instead we found 176 coded responses. Then, the percentage of each category of responses is given as follows.

Table 8. Frequency of teachers' performance on problem solving task

Type of responses	Sub-type	Codes	Task 1	Task 2	N	%	Total per type	%
Comprehension error	Misunderstanding the instruction	C-1	6	9	15	8.52%	37	21.02%
	Error in selecting information	C-2	11	11	22	12.50%		
Transformation/devising strategies error	Procedural tendency	T-1	7	4	11	6.25%	41	23.30%
	Taking too much account of the context	T-2	1	3	4	2.27%		
	Wrong mathematical operation/concept	T-3	9	17	26	14.77%		
Mathematical Processing error	Algebraic error	P-1	1	1	2	1.14%	8	4.55%
	Arithmetical error	P-2	0	3	3	1.70%		
	Measurement error	P-3	1	0	1	0.57%		
	Unfinished answer	P-4	1	1	2	1.14%		
Interpretation error		I	18	12	30	17.05%	30	17.05%
Full understanding	Arithmetical approach	F-1	22	10	32	18.18%	35	19.89%
	Algebraic approach	F-2	3	0	3	1.70%		
Unknown		U	11	14	25	14.20%	25	14.20%

Table 8 shows that transformation/devising strategies error constitute to the most frequently found from teachers' work (23.30%), while mathematical process error, conversely, become the least frequently observed (4.55%). Moreover, they also performed comprehension error highly, i.e., 21.03%. This point out that teachers found some difficulties in early stages of problem solving steps.

As examples on how teachers perform those errors, the following figures show comprehension error and transformation-interpretation error, respectively on task 1.

Jawab:

Skr antara km Mentari & km Rajawali. adalah 1-0

Jkni dia :

	M	K	
M & S :	1	1	:
S & R :	1	0	:
M & R :	1	0	:

Jawab:

$$\text{Mentari} : 1 + 0 + 0 = 1$$

$$\text{Surya FC} : 1 + 0 + 0 = 1$$

$$\text{Rajawali} : 0 - 2 + 0 = -2$$

$$\text{Mentari} : 7 - 5 = 2$$

$$\text{Surya FC} : 8 - 6 = 2$$

$$\text{Rajawali} : 5 - 9 = -4$$

$$\text{Mentari} - \text{Rajawali} : 3 - 6$$

a. C-2

b. T-1 and I

Figure 1. Examples of errors on task 1

On comprehension error, teacher at figure 1a was unable to distinguish between relevant and irrelevant information given from the table. He only considered information in column won, lost, and drawn without giving more attention to the column goals for and goals against to convey a calculation. Thus, we coded it as C-2. Meanwhile, transformation error was performed by teacher at figure 1b who tended to use directly a mathematical procedure, i.e. addition and subtraction without analyzing whether or not it was needed and did not provide precise mathematical argumentation supporting the procedure he used. Consequently, he got the score in negative number, which is not possible to happen in real world setting. Thus, we also coded it as I error.

Regarding transformation error and comprehension error on task 2, we give these following examples.

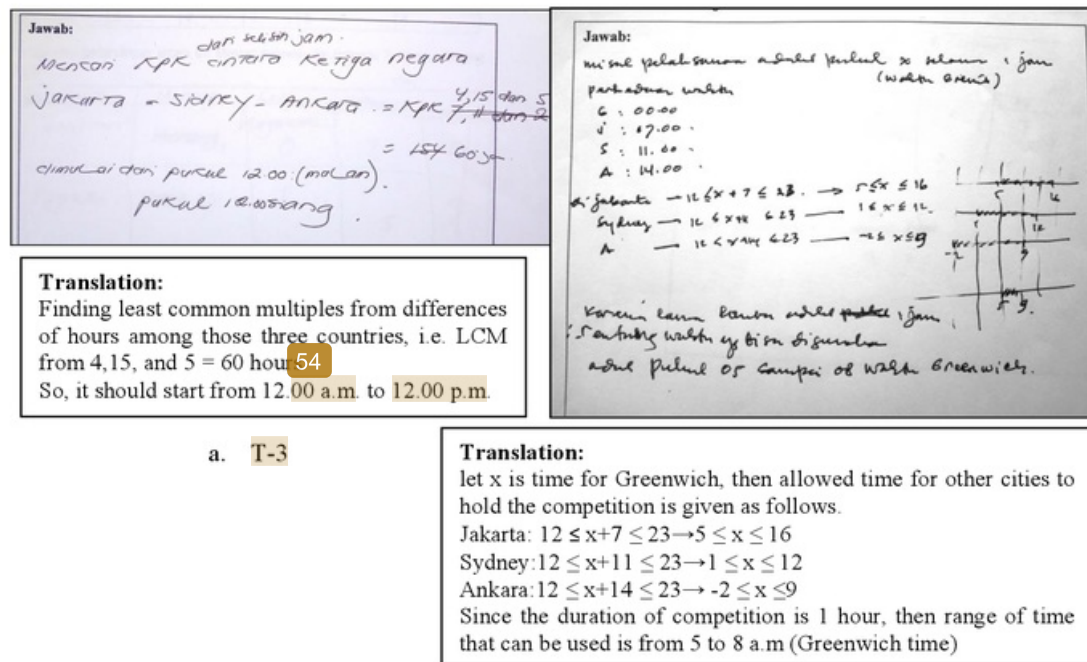


Figure 2. Examples of errors on task 2

Figure 2a was coded as transformation error because the teacher provided wrong mathematical model to find the best time chosen for holding the competition. She seemed trying to find LCM of the differences of hours from the three cities which is not suitable to find the solution. Hence, this error was coded as T-3. Meanwhile, figure 2b actually shows a unique strategy, i.e. using algebraic approach, which was not found from other teachers' work. However, the teacher did not perform carefully in selecting information related to model of inequality for Ankara time, i.e. writing $12 \leq x + 14 \leq 23$, instead of $12 \leq x + 2 \leq 23$. Thus, we coded it as P-1 since it contains algebraic error of finding solution of inequality.

Teachers' complete performance in solving the tasks is also interested to be discussed. Here, we found two approaches, i.e. arithmetical and algebraic approach. Here are examples of these approaches from task 1.

Jawab: Misal:
 MENTARI vs SURYA FC (12v1) = $a : a$
 MENTARI vs RAJAWALI = $b : c$
 SURYA FC vs RAJAWALI = $d : e$

Maka:
 Dari: MENTARI : $a + b = 7$ & $a + c = 5$
 SURYA FC : $a + d = 6$ & $a + e = 6$
 RAJAWALI : $c + e = 5$ & $b + d = 9$

Sehingga:
 $a + b = 7$ & $b = 7 - a$ & $a = 3$
 $a + d = 6$ & $d = 6 - a$ & $d = 3$
 $a + c = 5$ & $c = 5 - a$ & $c = 2$
 $c + e = 5$ & $e = 5 - c$ & $e = 3$
 $b + d = 9$ & $b = 9 - d$ & $b = 6$

Sehingga:
 $a = 3$ & $b = 6$ & $c = 2$ & $d = 3$ & $e = 3$

Maka: MENTARI vs RAJAWALI = $4 : 2$

Translation:

Let score of Mentari vs Surya FC = $a : a$

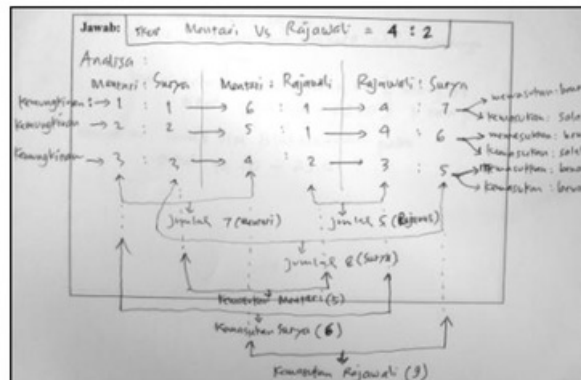
Mentari vs Rajawali = $b : c$

Surya FC vs Rajawali = $d : e$

Then

(algebraic operation same with shown figure)

So, Mentari vs Rajawali is $4 : 2$



Translation:

Analysis

	Mentari vs Surya	Mentari vs Rajawali	Rajawali vs Surya
possibility	1:1	6:1	4:7
possibility	2:2	5:1	4:6
possibility	3:3	4:2	3:5

a. F-1

b. F-2

Figure 3. Examples of teachers' complete performance on the tasks

Figure 3a shows that teacher used symbols representing number of goals successfully shot to another team, then carrying out a well-operated algebraic form by seeing about information of goals for and goals against from the table in task 1 to find out each of goals produced by each of teams. Interestingly, the teacher started to symbolize score of Mentari vs Surya by $a : a$, which means he know that number of goals produced between Mentari and Surya is same based on information of number of drawn match given in the table. Similarly, teacher at figure 3b also applied this information to find each of score between two teams of each match, but he applied arithmetical approach by listing some possibilities of score and then carrying out simple operation (addition) of number to find out the score of Mentari vs Surya FC.

4.2 Teachers' difficulties which regard to understand problem solving

We categorized teachers' difficulties into three type of questions, i.e. problem solving steps, designing problem solving task, and causes of difficulties in designing problem solving task. Table 7 show teachers' responses on this issue.

Table 9. Frequency of teachers' difficulties on problem solving

Questions	Options	The number of teacher choosing the options				Total	Percentage
		Private	%	State	%		
Most difficult steps of problem solving	read the task	4	16.00%	7	15.56%	11	15.71%
	understand problem	6	24.00%	18	40.00%	24	34.29%
	determine precise mathematical model and strategies	15	60.00%	20	44.44%	35	50.00%

Difficulties in designing problem solving task	employ mathematical procedures and facts	2	8.00%	2	4.44%	4	5.71%
	interpret result to look back to initial problem	4	16.00%	11	24.44%	15	21.43%
	determine real world context on mathematics topic	21	84.00%	29	64.44%	50	71.43%
	make correct written sentence	5	20.00%	7	15.56%	12	17.14%
	design task with more than one solution	2	8.00%	2	4.44%	4	5.71%
	design task with more than one alternative strategies	5	20.00%	8	17.78%	13	18.57%
	design task with more than one alternative solution and strategies	3	12.00%	9	20.00%	12	17.14%
	design simple task but challenging	16	64.00%	23	51.11%	39	55.71%
	not frequently solve higher order thinking-based tasks	9	36.00%	10	22.22%	19	27.14%
	never learn how to design problem solving task	10	40.00%	12	26.67%	22	31.43%
Causes of difficulties in designing problem solving task	frequently make set of evaluation only testing on calculation and procedure	6	24.00%	6	13.33%	12	17.14%
	not frequently read references about problem solving task	10	40.00%	18	40.00%	28	40.00%
	frequently use mathematical task provided in text book	5	20.00%	10	22.22%	15	21.43%
	too focus on guiding students in solving task using one step solution	10	40.00%	17	37.78%	27	38.57%
	difficulties regarding a variety of students' ability in classroom	2	8.00%	2	4.44%	4	5.71%

From the table, regarding steps of problem solving, teachers especially find difficulties mostly in step of determining strategies/mathematical model (50.00%), followed by understanding problem/comprehension (34.29%), and looking back at initial problem (21.43%). The difficulty in designing a problem-solving task, especially are determining the context of real problems of mathematical topics (71.43%) and designing simple task but challenging (55.71%). The top four of cause of these difficulties are almost evenly not frequently read references about problem solving task (40.00%), too focus on guiding students in solving task using one step solution (38.57%), never learn how to design problem solving task (31.43%), and not frequently solve higher order thinking-based tasks (27.14%).

4.3 Teachers' beliefs on mathematical problem solving

Belief in mathematics can be classified into three parts, namely mathematics as a tool, mathematics as body static and dynamic mathematics as human creations. Table 7 shows teachers' average score of beliefs on mathematics, teaching mathematics, and learning mathematics.

Table 10. Score of teachers' beliefs on mathematical problem solving

Type of beliefs	Private	State	Total	Interpretation
mathematics	2.35	2.02	2.14	body static/platonist
teaching mathematics	2.29	2.41	2.36	problem solving
learning mathematics	1.93	1.89	1.90	body static/platonist

In general, the table above shows that teachers still tend to view mathematics, teaching mathematics, and learning mathematics as body static to problem solving which agrees to platonist-problem solving view as indicated score range varying from 1.90 - 2.36. However, belief in learning mathematics has the lowest score compared to the other two beliefs. Moreover, its score, i.e. 1.90, is rather closer to scoring range for tool view. Hence, teachers' belief on learning mathematics is rather instrumentalist. Supporting this data, by calculating percentage of frequency of teacher choosing 'instrumentalist' option, for instance, we noted that most teacher (38 out of 70) agree that students should learn mathematics by starting to work on simple questions to harder questions. On the other hand, in viewing mathematics, for instance, 38 out of 70 view mathematics as only tool to learn and develop other disciplines, while in viewing teaching mathematics, we noted that there were many teachers (22 out of 70 teachers) still believe that teaching mathematics should explain topic systematically followed by giving some examples and asking students to work on some questions. However, in viewing teaching mathematics, teachers relatively agree to apply problem solving/dynamic approach, as indicated by the highest score among three types of beliefs shown in table 7, i.e. 2.36. As an example, we noted that most teacher (47 out of 70) agree that teaching mathematics should start from posing contextual mathematics problem.

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5. Discussion and Conclusion

Results of studying teachers' understanding show that most teachers were likely found difficulties on problem solving content knowledge, especially in knowing problem solving strategy. Analysis of their performance in solving problem solving task also reveal that errors mostly happened when they start to devise strategies or precise mathematical model. These findings were then supported by their opinion that the most difficult steps of carrying out problem solving step is devising strategies. Moreover, understanding this knowledge needs likely more attention to deeply comprehend interrelated content knowledge regarding certain topic. For instance, when investigating prospective elementary teachers' common thinking about open-ended, Chapman [35] found that they actually know that open ended problem meant more than one answer but they were doubt about what this meant mathematically such as inability to provide example of other answers on a given open-ended task.

Regarding teachers' belief, the fact that teachers tend to view both mathematics and how students should learn mathematics as body static perspective, while they tend to believe to apply idea of problem solving as dynamic approach when teaching mathematics shows that belief in mathematics is not the only factor affecting the practice of teaching and the views of the students who are learning. Raymond [in 4] describes other factors besides belief in mathematics as teacher education programs, teaching social norms, teachers' life outside the classroom, characteristics of teacher's personality, the situation in the classroom, student life outside the classroom.

To sum up, we argue that: (1) Teachers' understanding on problem solving content knowledge was less than those on pedagogical content knowledge (2) Teachers more believed on mathematics and mathematics learning as body static, while in practice, they tend believe to the views that they should teach mathematics as a dynamic/problem solving activity.

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Appendix

Task 1

Futsal Tournament

In the end of this year, a subdistrict government held a futsal tournament involving some futsal teams from schools within the subdistrict. To prepare the tournaments, three futsal teams from Eksakta school held a futsal training in their schoolyard once a week. Each of them face each other in a match exactly once. The following table presents the result of the training this week.

Team	Won	Lost	Drawn	Goals	
				Goals For	Goals Against
Mentari	1	0	1	7	5
Surya FC	1	0	1	8	6
Rajawali	0	2	0	5	9

Note:

Won :The number of matches won this week

Lost :The number of matches lost this week

Drawn :The number of matches drawn this week

Goals for :The number of goals scored this week

Goals against :The number of goals conceded this week

What is the score of the match Mentari vs Rajawali? Explain your reason.

Task 2

World Online Math Literacy Contest

An online Maths Literacy Competition was attended by participants from several cities, namely, Jakarta (Indonesia), Sydney (Australia) and Ankara (Turkey). The competition lasts for 1 hour at the same time on each of the local time. Here is the time difference in each country



Greenwich (midnight)



Ankara (early morning)



Jakarta (morning)



Sydney (noon)

Since all participants are students, so they can not follow the contest at 07:00 a.m. to 12:00 a.m. on each local time for having school and at 11 p.m. to 6:00 a.m. for bedtime. Find the appropriate range of time that the committee should select for the contest. Explain your strategy.

An Investigation of Secondary Teachers' Understanding and Belief on Mathematical Problem Solving

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